Cyclic Correlation-Based Cooperative Detection for OFDM-Based Primary Users

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Abstract

This paper develops a new robust cyclostationary detection technique for spectrum sensing of OFDM-based primary users (PUs). To do so, an asymptotically constant false alarm rate (CFAR) multi-cycle detector is proposed and its statistical behavior under null hypothesis is investigated. Furthermore, to achieve higher detection capability, a soft decision fusion rule for performing cooperative spectrum sensing (CSS) in secondary networks is established. The proposed CSS scheme aims to maximize the deflection criterion at the fusion center (FC), while the reporting channels are under Rayleigh fading. In order to be able to evaluate the performance of the cooperative detector, some analytic threshold approximation methods are provided for the cases where the FC has direct sensing capability or not. Through numerical simulations, the proposed local and CSS schemes are shown to significantly enhance CR network performance in terms of detection probability metric.

1. Introduction

Cooperative spectrum sensing (CSS) methods were proposed in literature to improve the detection performance of secondary networks [1, 2]. However, most existing scenarios assume that all the cooperating users use the commonly adopted energy detection (ED) technique for their local sensing. But, despite the simplicity, quickness and no requirements of pre-knowledge about the PU's signal, the ED method has some challenging issues. For example, it cannot differentiate primary users (PUs) from secondary users (SUs), and requires knowing the noise variance to ensure proper detection performance [3]. Therefore, ED-based CSS methods are prone to false detections [2, 4].

In addition, ED-based soft decision fusion rules that has been recently introduced in literature, such as [5-7], assume that the perfect knowledge about the noise variances of sensing channels are available at fusion center (FC). However, in practice, this assumption may be unrealistic. Therefore, the cooperative sensing methods based on local ED may be very susceptible to noise uncertainties of sensing channels and therefore their performance can be dictated by the accuracy of the noise power estimations at SUs.

Cyclostationary detection (CD)-based sensing methods are proposed in literature to address the above issues [2,4,8-11]. These detectors exploit inherent cyclostationary properties of digitally-modulated signals and have acceptable performance in very low SNRs [2,12]. However, despite the advantages of CD over ED-based spectrum sensing, there are rather limited researches on CD-based CSS, mostly because of its complicated analytic expressions and also complexities that may arise in implementation of the CD algorithms.

Some recent works were addressed the complexity issue of CD methods. For example, a new CD-based CSS method is proposed in [13]. But, there are two drawbacks with this method. Firstly, its performance can be dictated by the uncertainties in estimating the noise variances of sensing channels at the SUs since the thresholds for the single-cycle and multi-cycle detectors are functions of the noise variance. Secondly, because the proposed detectors are constructed based on the cyclic autocorrelation function (CAF) estimates at zero time-lags, the performance for detecting OFDM-based primary or secondary users may be very poor. It is well-known that the strong cyclic frequencies of OFDM-based transmissions are located at time-lags equal to $\tau = T_u$, where T_u is the useful symbol length of the OFDM symbol [11].

In this paper, we propose a simplified cyclic correlation-based detection algorithm for the

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local spectrum sensing which does not require any specific assumption about the distribution of PU signals. In other words, to combat the above-mentioned spectrum sensing problems (that exist in conventional ED and some CD methods), we propose a CD detector that does not:

- require any specific assumption about the statistical distribution of PU signal,
- need any prior knowledge about the noise variance of the sensing channel,
- know the statistics of fading channel.

In this context, we derive a Generalized Likelihood Ratio Test (GLRT) based on asymptotic distribution of second-order cyclic correlation vector. Then, we analytically characterize the asymptotic distributions of the test statistic under null and alternative hypotheses. In order to evaluate the performance of the test, we derive closed-form expressions for the false alarm and detection probabilities, and verify them through numerical simulations.

It is widely-known that using multiple cycle frequencies at cyclostationary detectors does improve the detection performance. Thus, we also propose a new multi-cycle detector.

Based on the proposed local sensing method, we then develop a weighted soft combination method for the CSS. This method is based on the deflection criterion maximization at the FC and achieves better detection performance compared to the conventional cooperative detectors. Furthermore, it provides reliable detection performance when both the reporting and sensing channels are under fading impairments.

In this paper, a threshold estimation method for CSS is provided in order to perform decision making at the fusion center. Simulation results confirm that the proposed analytical threshold setting procedures have adequate accuracy for performance analysis purposes. It should be noted that our proposed method does not need any prior knowledge about the noise variances of sensing channels or their fading statistics.

The remainder of this paper is organized as follows. System model is presented in Section 2. The local sensing strategy is described in Section 3. The cooperative detection algorithm is developed in Section 4. Performances of proposed schemes are investigated in Section 5. Finally, the conclusions are drawn in Section 6.

2. System Model

It is assumed that the base-band discrete-time received signal for ith SU, $x_i(n)$, $i = 1,2,\cdots,L$, at a time instance n is given by:

$$x_i(n) = \eta\, g_i\, \tilde{x}(n) + w_i(n), \qquad n = 1,\cdots,M, \qquad (1)$$

where L refers to the number of secondary users existing in the network, g_i denotes the channel fading coefficient between PU to ith SU, and $w(n) \sim \mathcal{CN}(0, \delta_i^2)$ with δ_i^2 as the variance of the complex additive Gaussian noise. Note that δ_i^2 and g_i are generally unknown.

Moreover, $\eta = 0$ and $\eta = 1$ correspond to null (inactive PU) and alternative (active PU) hypotheses, respectively. We assume that the PU is either active or inactive during the sensing duration. The signal transmitted by PU is denoted by $\tilde{x}(n)$. Without loss of generality, $\tilde{x}(n)$, g_i and $w_i(n)$ are assumed to be independent of each other. Furthermore, conditional independence of spatially distributed SUs is assumed [2].

After the decision statistic $\mathcal{T}_i$ at the ith SU is computed, it is transmitted to the FC through an independent reporting channel that experiences fading. Hence,

$$\tilde{\mathcal{T}}_i = h_i \mathcal{T}_i + z_i, \qquad i = 1,\cdots,L, \qquad (2)$$

where $z_i \sim \mathcal{N}(0, \sigma_i^2)$ and h_i is a real-valued fading envelope with $h_i > 0$. We assume that $\{h_i\}_{i=1}^{L}$ are constant during the detection interval. Without loss of generality, we assume that the Rayleigh fading channels have unit powers (i.e. $E(h_i^2) = 1$). In addition, the above model assumes the well-known phase-coherent reception at FC. Note that $\tilde{\mathcal{T}}_i$ represents the received signal from ith SU.

Since in many cognitive radio scenarios the envelope of the fading channel and the noise variance can be estimated in advance, we assume that the quantities $\{h_i\}_{i=1}^{L}$ and $\{\sigma_i^2\}_{i=1}^{L}$ are perfectly known to the FC [5-7, 14, 15].

3. Proposed Cyclostationarity-based Detection Method

Assume that we want to test for the presence of the cyclostationarity at a candidate cycle frequency α (known prior or can be estimated [12]) in the received signal $x(n)$.

For a given time lag τ and a cycle frequency α, the estimated cyclic autocorrelation function (CAF) is defined to be [12]:

$$\widehat{\mathcal{M}}_{2x}(\alpha; \tau) \triangleq \hat{E}_M\{x(n)x^*(n+\tau)e^{-j2\pi n\alpha}\}, \qquad (3)$$

where $\hat{E}_M\{f(n)\} \triangleq (\frac{1}{M}).\sum_{n=1}^{M} f(n)$ denotes the M-sample average. It has been proven in [16] that subject to certain mixing conditions, $\widehat{\mathcal{M}}_{2x}(\alpha; \tau)$ is a consistent and asymptotically (i.e. as $M \to \infty$) normal estimator of the cyclic moment $\mathcal{M}_{2x}(\alpha; \tau)$. In essence, the expression $\sqrt{M}[\widehat{\mathcal{M}}_{2x}(\alpha; \tau) - \mathcal{M}_{2x}(\alpha; \tau)]$ asymptotically converges in distribution to a complex normal

variable [16]. Therefore, the real and imaginary parts of $\widehat{\mathcal{M}}_{2x}(\alpha;\tau)$ are jointly Gaussian.

In this paper, we propose to only consider the real part of $\widehat{\mathcal{M}}_{2x}(\alpha;\tau)$. This causes the resulting test statistic to have a reduced complexity, as compared to the case where the whole structure is considered for deriving the decision statistic. Let,

$$\widehat{\mathcal{M}}_{2x}(\alpha;\tau) \triangleq \Re\{\widehat{\mathcal{M}}_{2x}(\alpha;\tau)\} + j\Im\{\widehat{\mathcal{M}}_{2x}(\alpha;\tau)\} \qquad (4)$$

where $\Re$ and $\Im$ denote the real and imaginary parts, respectively. Consequently, the following convergence holds true for the asymptotic case:

$$\sqrt{M}[\Re\{\widehat{\mathcal{M}}_{2x}(\alpha;\tau)\} - \Re\{\mathcal{M}_{2x}(\alpha;\tau)\}] \xrightarrow{L} \mathcal{N}(0,\varrho^2) \quad (5)$$

Our aim is to devise a cyclic feature detector based on the above property. To this end, let us define

$$\mathcal{U} \triangleq \sqrt{M}\,\Re\{\mathcal{M}_{2x}(\alpha;\tau)\}, \qquad (6)$$

and its estimation:

$$\widehat{\mathcal{U}} \triangleq \sqrt{M}\,\Re\{\widehat{\mathcal{M}}_{2x}(\alpha;\tau)\}. \qquad (7)$$

Following the discussions in [16], it can be shown that:

$$\widehat{\mathcal{U}} \xrightarrow{L} \begin{cases} \mathcal{N}(0,\varrho^2), & \text{under } \mathcal{H}_0 \\ \mathcal{N}(\mu_1,\varrho^2), & \text{under } \mathcal{H}_1 \end{cases}, \qquad (8)$$

where $\mu_1 = \mathcal{U}$, which is unknown but is non-random. Based on above discussions we can constitute the following generalized likelihood ratio test (GLRT):

$$\mathcal{L}_G = \frac{\Pr(\widehat{\mathcal{U}};\hat{\mu}_1,\hat{\varrho}^2|\mathcal{H}_1)}{\Pr(\widehat{\mathcal{U}};\hat{\varrho}^2|\mathcal{H}_0)} = \frac{\frac{1}{\sqrt{2\pi\hat{\varrho}^2}}\exp(-\frac{(\widehat{\mathcal{U}}-\widehat{\mathcal{U}})^2}{2\hat{\varrho}^2})}{\frac{1}{\sqrt{2\pi\hat{\varrho}^2}}\exp(-\frac{\widehat{\mathcal{U}}^2}{2\hat{\varrho}^2})}. \qquad (9)$$

In the above equation, the numerator is obtained by substituting μ_1 with its estimation. Thus, the decision statistic can be computed from the generalized log-likelihood ratio (GLLR) function:

$$\mathcal{T}_G = 2\ln\mathcal{L}_G = \frac{\widehat{\mathcal{U}}^2}{\hat{\varrho}^2} \begin{cases} \leq \eta: & \text{Decide } \mathcal{H}_0 \\ \geq \eta: & \text{Decide } \mathcal{H}_1 \end{cases} \qquad (10)$$

where η is the threshold value. Since $\widehat{\mathcal{M}}_{2x}(\alpha;\tau)$ is mean-square sense consistent [16], we can obtain that

$$\lim_{M\to\infty} \widehat{\mathcal{U}}_{2x}(\alpha;\tau) \xrightarrow{\text{m.s.s.}} \mathcal{U}_{2x}(\alpha;\tau) \qquad (11)$$

where denotes the mean-square sense convergence. Therefore, under null hypothesis,

$$\lim_{M\to\infty} \widehat{\mathcal{U}}_{2x}(\alpha;\tau) \xrightarrow{L} \mathcal{Z} \sim \mathcal{N}(0;\varrho^2) \qquad (12)$$

Furthermore, $\hat{\varrho}^2$ converges in the m.s.s. to ϱ^2 [12, 16]. Since convergence in m.s.s. implies convergence in probability [p. 234][17], we can conclude that $\lim_{M\to\infty} \hat{\varrho}^2 \xrightarrow{P} \varrho^2$. Therefore, based on Slutsky's theorem [Th.3.3][18], we deduce that

$$\lim_{M\to\infty} \frac{\widehat{\mathcal{U}}^2}{\hat{\varrho}^2} \xrightarrow{L} \frac{1}{\varrho^2}\mathcal{Z}^2. \qquad (13)$$

Consequently, under $\mathcal{H}_0$ the following result can be obtained:

$$\lim_{M\to\infty} \mathcal{T}_G \xrightarrow{L} \chi_1^2, \qquad (14)$$

and under $\mathcal{H}_1$ we have

$$\lim_{M\to\infty} \mathcal{T}_G \xrightarrow{L} \acute{\chi}_1^2\left(\frac{\mathcal{U}^2}{\varrho^2}\right). \qquad (15)$$

Hence, once the threshold is fixed for a given P_{fa}, the obtained P_d will depend naturally on the unknown parameter $\hat{\varrho}^2$. Therefore, in practice, we propose to use the following approximate distribution for large values of M:

$$\mathcal{T}_G \sim \acute{\chi}_1^2\left(\frac{\widehat{\mathcal{U}}^2}{\hat{\varrho}^2}\right). \qquad (16)$$

The variance ϱ^2 can be computed by the following expression:

$$\begin{aligned} \varrho^2 &= \text{Var}\{\widehat{\mathcal{U}}\} \\ &= \text{Mcum}\left\{\Re\{\widehat{\mathcal{M}}_{2x}(\alpha;\tau)\}, \Re\{\widehat{\mathcal{M}}_{2x}(\alpha;\tau)\}\right\} \\ &= \frac{1}{2}\Re\left\{\text{M}\mathbb{C}\text{um}\{\widehat{\mathcal{M}}_{2x}(\alpha;\tau)\} \right. \\ &\qquad\qquad \left. + \text{M}\mathbb{C}\text{um}\{\widehat{\mathcal{M}}_{2x}^*(\alpha;\tau)\}\right\}. \end{aligned} \qquad (17)$$

where $\mathbb{C}\text{um}\{.,.\}$ denotes the cumulant operator. Therefore, if we define two asymptotic covariance:

$$\mathcal{Q} \triangleq \text{M}\mathbb{C}\text{um}\{\widehat{\mathcal{M}}_{2x}(\alpha;\tau), \widehat{\mathcal{M}}_{2x}(\alpha;\tau)\}, \qquad (18)$$

and

$$\mathcal{S} \triangleq \text{M}\mathbb{C}\text{um}\{\widehat{\mathcal{M}}_{2x}(\alpha;\tau), \widehat{\mathcal{M}}_{2x}^*(\alpha;\tau)\}, \qquad (19)$$

then, the asymptotic variance of $\widehat{\mathcal{U}}$ can be expressed as:

$$\varrho^2 = \frac{1}{2}\Re\{\mathcal{S} + \mathcal{Q}\}. \qquad (20)$$

In practice, these elements can be estimated respectively by:

$$\hat{\mathcal{Q}} \triangleq \frac{M}{P}\sum_{s=-\frac{P-1}{2}}^{\frac{P-1}{2}} W_P(s)\widehat{\mathcal{M}}_{2x}\left(\alpha - \frac{s}{M};\tau\right)\widehat{\mathcal{M}}_{2x}\left(\alpha + \frac{s}{M};\tau\right), \qquad (21)$$

and

$$\hat{\mathcal{S}} \triangleq \frac{M}{P}\sum_{s=-\frac{P-1}{2}}^{\frac{P-1}{2}} W_P(s)\left|\widehat{\mathcal{M}}_{2x}\left(\alpha + \frac{s}{M};\tau\right)\right|^2. \qquad (22)$$

In the above equation, W_P is a normalized smoothing window with an odd length P,

$$W_P(s) \triangleq \begin{cases} I_0\left(\beta\sqrt{1 - \frac{4s^2}{P^2}}\right), & -\frac{P-1}{2} \leq s \leq \frac{P+1}{2}, \\ 0, & \text{o.w.} \end{cases} \qquad (23)$$

where $I_0(.)$ is the modified Bessel function of first-kind and zero-order. It should be noted that the values of P and β should be pre-set in the detection algorithm.

Using the above results, the estimated variance can be expressed as $\hat{\varrho}^2 = \frac{1}{2}[\Re\{\hat{\mathcal{Q}}\} + \hat{\mathcal{S}}]$. However, in the low-SNR regimes, which is of interest in spectrum sensing scenarios as well as cyclostationarity-based detection applications, we have $\hat{\mathcal{S}} \gg \Re\{\hat{\mathcal{Q}}\}$. Thus, in practice, we propose the following expression for the variance estimation:

$$\hat{\varrho}^2 \approx \frac{M}{2P} \sum_{s=-\frac{P-1}{2}}^{\frac{P-1}{2}} W_P(s) \left| \hat{\mathcal{M}}_{2x} \left(\alpha + \frac{s}{M}; \tau \right) \right|^2. \tag{24}$$

Since this expression is derived for low-SNR conditions, it may not provide enough accuracy for moderate- and high-SNR regimes. However, we will show in Section II through simulation results that (24) results in an acceptable performance for all SNR values.

3.1 Threshold Selection and Analytic Performance Expression

Using (14) we can obtain the false alarm probability as below:

$$P_{fa} = \Pr\{\mathcal{T}_G > \eta | \mathcal{H}_0\} = 1 - \mathcal{F}_{\chi_1^2}(\eta)$$
$$= 1 - \frac{\gamma\left(\frac{1}{2}, \frac{\eta}{2}\right)}{\Gamma\left(\frac{1}{2}\right)}, \tag{25}$$

where $\mathcal{F}_{\chi_1^2}$ is cumulative distribution function (CDF) of chi-square variable with one degrees of freedom. In addition, $\gamma(.,.)$ and $\Gamma(.)$ denote the lower incomplete Gamma function and Gamma function, respectively [19]. It can be shown that

$$\gamma\left(\frac{1}{2}, \frac{\eta}{2}\right) = \sqrt{\pi}. \, \mathrm{erf}\left(\sqrt{\eta/2}\right) = 1 - 2Q\left(\sqrt{\eta}\right), \tag{26}$$

where

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2)\, dt, \tag{27}$$

and Q denotes the right-tail probability of standard Gaussian distribution [19]. Therefore, the threshold value can be determined by

$$P_{fa} = 2Q\left(\sqrt{\eta}\right). \tag{28}$$

Since the threshold value does not depend on the sensing channel parameters (e.g. SNR value, noise variance, fading gain, etc.), it can be computed directly from the P_{fa} value, and therefore the proposed detection algorithm can be introduced as a constant false alarm rate (CFAR) test.

Using the same line as above, the detection probability can be derived as follows:

$$P_d = \Pr\{\mathcal{T}_G > \eta | \mathcal{H}_1\} = 1 - \mathcal{F}_{\hat{\chi}_1^2(\lambda)}(\eta)$$
$$= 1 - \mathcal{F}_{\hat{\chi}_1^2(\lambda)}\left(\left(Q^{-1}(0.5 P_{fa})\right)^2\right), \tag{29}$$

where

$$\lambda \triangleq \frac{\hat{\mathcal{U}}^2}{\hat{\varrho}^2}, \tag{30}$$

Validity of the above asymptotic distributions is confirmed in our subsequent discussions in Section 5.

3.2 Extension to the Multi-Cycle Detectors

It is well-known that detecting multiple cyclic frequencies at the same time would enhance the detection performance [2, 13, 20].

Most of the current researches have simply proposed the summation of test statistics of different cyclic frequencies for multi-cycle detection purposes [2, 8, 13]. However, using the same procedure as in [20], we alternatively propose the following multi-cycle (MC) detector:

$$\mathcal{T}_{MC} = \frac{1}{\sqrt{\sum_{i=1}^{N_\alpha} w_i^2}} \sum_{i=1}^{N_\alpha} w_i^2 \mathcal{T}_G^{\alpha_i}, \tag{31}$$

where

$$w_i \triangleq \left\{ \frac{\mathcal{U}^2}{\varrho^2} \right\} \Bigg|_{\alpha_i}, \tag{32}$$

and $\mathcal{T}_G^{\alpha_i}$ denote the noncentrality parameter and the test statistic corresponding to a cyclic frequency α_i, respectively. In the same equation, N_α denotes the number of intended cyclic frequencies. It is noteworthy that the above detector combines different cyclic frequencies in a way that the deflection coefficient is maximized [20].

To perform hypothesis testing, the null distribution of (31) is required to be computed. Since CAFs of different cyclic frequencies are statistically independent under the null hypothesis [8], we can approximate the CDF by inversion of corresponding characteristic function (CF) of $\mathcal{T}_{MC}$. Fist, assume that the CF of $\mathcal{T}_{MC}$ is defined as:

$$\Phi_i(t) \triangleq E\left[\exp(jt\mathcal{T}_G^{\alpha_i})\right]. \tag{33}$$

In this case, we can write

$$\Phi_{\mathcal{T}_{MC}}(t) = \prod_{i=1}^{N_\alpha} \Phi_i(w_i t). \tag{34}$$

Since $\mathcal{T}_G^{\alpha_i} \sim \chi_1^2$, the following equation holds true:

$$\Phi_i(t) = (1 - 2jt)^{-1/2}. \tag{35}$$

Using the above expression, we can conclude that:

$$\Phi_{\mathcal{T}_{MC}}(t) = \prod_{i=1}^{N_\alpha} (1 - 2jw_i t)^{-1/2}. \tag{36}$$

Now, employing the well-known Gil-Palaez theorem [21], we can compute the corresponding CDF of $\Phi_{\mathcal{T}_{MC}}$ under $\mathcal{H}_0$ as follows:

$$\mathcal{F}_{\mathcal{T}_{MC}}(x) = \frac{1}{2} - \frac{1}{\pi} \int_0^\infty \frac{\Im\{\Phi_{\mathcal{T}_{MC}}(t)e^{-jtx}\}}{t} dt. \tag{37}$$

Consequently, substitution of (36) into (37) yields the resultant CDF expression:

$$\mathcal{F}_{\mathcal{T}_{MC}|\mathcal{H}_0}(x)$$
$$= \frac{1}{2} - \frac{1}{\pi} \int_0^\infty \frac{\Im\left\{ \prod_{i=1}^{N_\alpha} (1 - 2jw_i t)^{-\frac{1}{2}} e^{-jtx} \right\}}{t} dt, \tag{38}$$

which can be simply computed using the numerical integration techniques. Finally, since

$$P_{fa} = \Pr\{\mathcal{T}_{MC} > \breve{\eta} | \mathcal{H}_0\} = 1 - \mathcal{F}_{\mathcal{T}_{MC}}(\breve{\eta}), \tag{39}$$

we can calculate the threshold value as below:
$$\breve{\eta} = \mathcal{F}_{\mathcal{T}_{MC}}^{-1}(1 - P_{fa}). \qquad (40)$$

Accuracy of the proposed method is investigated in Fig. 1. We have simulated the distribution function of sum of five central chi-square variables as $0.4927\chi_1^2 + 0.5478\chi_2^2 + 0.0768\chi_4^2 + 0.5523\chi_6^2 + 0.3824\chi_2^2$. The weights are generated in a random manner. The simulation curve presents the real accurate distribution. As we can see, the estimated CDF, which is computed from (38), follows very closely the simulated distribution. Thus, the proposed threshold determination method can be reliably implemented at SUs.

Fig. 2 provides another example. The sum of ten random variables is simulated. The sum distribution is selected to be $0.4507\chi_2^2 + 0.2453\chi_3^2 + 0.3899\chi_7^2 + 0.1190\chi_1^2 + 0.4874\chi_5^2 + 0.0337\chi_4^2 + 0.4693\chi_6^2 + 0.2782\chi_2^2 + 0.0779\chi_7^2 + 0.1676\chi_1^2$. The results of numerical simulations confirms the superior performance of the proposed method.
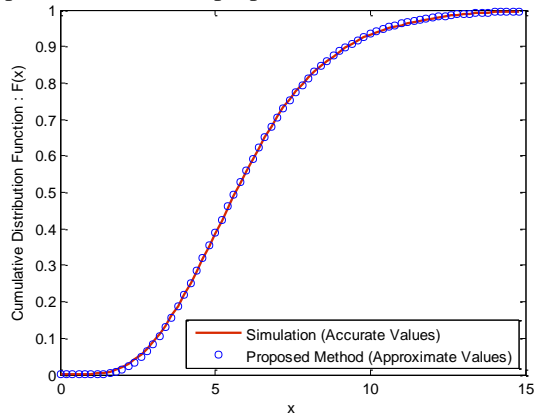

Fig. 1 Accuracy of the proposed threshold selection method. The simulated distribution is $0.4927\chi_1^2 + 0.5478\chi_2^2 + 0.0768\chi_4^2 + 0.5523\chi_6^2 + 0.3824\chi_2^2$ (random-generated weights).
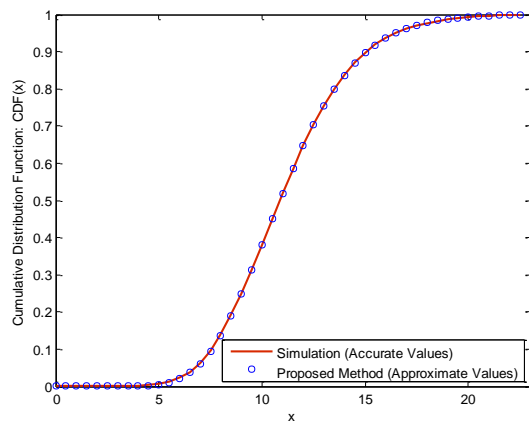

Fig. 2 Accuracy of the proposed threshold selection method. The simulated distribution is $0.4507\chi_2^2 + 0.2453\chi_3^2 + 0.3899\chi_7^2 + 0.1190\chi_1^2 + 0.4874\chi_5^2 + 0.0337\chi_4^2 + 0.4693\chi_6^2 + 0.2782\chi_2^2 + 0.0779\chi_7^2 + 0.1676\chi_1^2$. (random-generated weights).

4. Proposed Cooperative Spectrum Sensing Method

In this paper, we only consider the soft combination-based cooperative spectrum sensing method. We propose that the fusion center has the capability to directly sense the radio frequency spectrum. As we will confirm by simulation results, this improves the reliability of the final decision. Furthermore, we propose to employ a weighted combination fusion rule for fusing the soft decisions transmitted by SUs. Therefore, the proposed global decision rule at the FC can be expressed as:

$$\tilde{y} = w_0 \mathcal{T}_{G,0} + \sum_{i=1}^{L} w_i \tilde{\mathcal{T}}_{G,i} = w\Gamma \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \tilde{\eta}, \qquad (41)$$

where

$$w = [w_0, w_1, w_2, \cdots, w_L]^T, \qquad w_i \geq 0 \qquad (42)$$

is the weight vector used to build the weighted fusion rule, and

$$\Gamma = [\mathcal{T}_{G,0}, \tilde{\mathcal{T}}_{G,1}, \tilde{\mathcal{T}}_{G,2}, \cdots, \tilde{\mathcal{T}}_{G,L}]^T \qquad (43)$$

denotes the vector of observations. Note that w_0 corresponds to the weight that is assigned to the FC and the others are the corresponding weights of SUs.

4.1 Weight Vector Computation

The problem that we are encountering is how we should calculate the weight vector w, thereby the resultant detector achieves the best performance. To this end, we choose the deflection coefficient [22] as the performance metric.

In order to compute the deflection coefficient, we require the first and second-order moments of the test statistic (41). After some manipulation, we obtain the following expressions:

$$E_0[\tilde{y}] = w_0 + \sum_{i=1}^{L} w_i h_i = w^T h$$

$$Var_0[\tilde{y}] = 2w_0^2 + \sum_{i=1}^{L} w_i^2 (2h_i^2 + \sigma_i^2) = w^T m_0 w$$

$$E_1[\tilde{y}] = w_0(1 + \lambda_0) + \sum_{i=1}^{L} w_i h_i (1 + \lambda_i)$$
$$= w^T(h + H\lambda)$$

$$Var_1[\tilde{y}] = 2w_0^2(1 + 2\lambda_0) + \sum_{i=1}^{L} w_i^2 (2h_i^2(1 + 2\lambda_i) + \sigma_i^2)$$
$$= w^T m_1 w$$
$$(44)$$

where $E_0[.]$ ($Var_0[.]$) and $E_1[.]$ ($Var_1[.]$) denote the statistical expectation (variance) under null and alternative hypotheses, respectively. In the above equations,

$$\lambda = [\lambda_0, \lambda_1, \lambda_2, \cdots, \lambda_L]^T \qquad (45)$$

denotes the vector of estimated noncentrality parameters at different SUs, where $\lambda_i = \hat{\mathcal{U}}_i^2 / \hat{\mathcal{Q}}_i^2$.

Furthermore, the following definitions are supposed:

$h = [1, h_1, h_2, \cdots, h_L]^T$,

$m_0 = 2H^2 + \text{diag}(\sigma)$,

$\sigma = [0, \sigma_1^2, \sigma_2^2, \cdots, \sigma_L^2]^T$

$H = \text{diag}(1, h_1, h_2, \cdots, h_L)$,

$m_1 = 2H^2[I_{L+1} + 2\text{diag}(\lambda)] + \text{diag}(\sigma)$.

$$(46)$$

After some manipulation, we can obtain the deflection of cooperative detector as:

$$\mathcal{D}(w) \triangleq \frac{(E_1[\tilde{y}] - E_0[\tilde{y}])^2}{\text{Var}_0[\tilde{y}]} = \frac{(w^T H \lambda)^2}{w^T m_0 w}. \qquad (47)$$

We rewrite (45) and define the optimal weighting vector w_{DC} as the one that meets the following optimization problem:

$$w_{DC} = \underset{w \geq 0, \|w\|_2 = 1}{\text{argmax}} \frac{w^T \tilde{m} w}{w^T m_0 w} \ , \qquad (48)$$

where

$$\tilde{m} \triangleq H \lambda \lambda^T h^T, \qquad (49)$$

and $\|.\|_2$ denotes the Euclidian norm (i.e. $\|w\|_2 = \left(\sum_{i=0}^L w_i^2\right)^{1/2}$). In order to achieve a unique solution for the optimization problem, we have confined the weight vector to have a unit norm.

If $m_0^{-1/2}$ be the square root obtained from the Cholesky decomposition of m_0, substituting $w = m_0^{-T/2} v$, we can get

$$\mathcal{D}(v) = \frac{v^T \left(m_0^{-1/2} H \lambda \lambda^T H^T m_0^{-T/2}\right) v}{v^T v}, \qquad (50)$$

which is in the Rayleigh's quotient form [23]. Hence, $v = m_0^{-1/2} H \lambda$ and normalizing the result gives the optimal weight vector as:

$$w_{DC} = \frac{m_0^{-T/2} m_0^{-1/2} H \lambda}{\left\| m_0^{-T/2} m_0^{-1/2} H \lambda \right\|_2}. \qquad (51)$$

It should be note that the suboptimal linear sum-detector is obtained by substituting $w_{EGC} = [1, 1, \cdots, 1]^T/\sqrt{L}$ into (41).

4.2 Threshold Selection at Fusion Center

4.2.1 The case that FC does not have direct observation

According to the central limit theorem [24], if the number of SUs in network is large enough (in practice, greater than or equal to 10), we can write

$$\tilde{y} \sim \begin{cases} \mathcal{N}(E_0[\tilde{y}], \text{Var}_0[\tilde{y}]), & \text{under } \mathcal{H}_0 \\ \mathcal{N}(E_1[\tilde{y}], \text{Var}_1[\tilde{y}]), & \text{under } \mathcal{H}_1 \end{cases} \qquad (52)$$

Based on above distributions, we can find the decision threshold in (41), or equivalently, the false alarm probability:

$$\tilde{\eta} \simeq \sqrt{w^T m_0 w} Q^{-1}(P_{fa}) + w^T h \qquad (53)$$

The accuracy of the above distribution is subsequently examined by Monte-Carlo simulations.

4.2.2 The case that FC has direct observation

In this section, we introduce a method to estimate the decision threshold in (41) for a given probability of false alarm. Since an analytical closed-form expression for the null distribution of the test statistic does not exist, we propose to approximate the distribution function by the numerical inversion of corresponding characteristic function (CF).

Under null hypothesis, the CF of a random variable X is $\Phi_{X|\mathcal{H}_0}(t) \triangleq E[\exp(jtX)|\mathcal{H}_0]$. From (2), the CF of received statistic can be obtained as ($i = 1, \cdots, L$)

$$\Phi_{\tilde{\mathcal{T}}_{G,i}|\mathcal{H}_0}(t) = (1 - 2jh_i t)^{-\frac{1}{2}} \exp\left(-\frac{\sigma_i^2 t^2}{2}\right). \qquad (54)$$

Applying the fact that

$$\Phi_{\tilde{y}|\mathcal{H}_0}(t) = \Phi_{\mathcal{T}_{G,0}|\mathcal{H}_0}(w_0 t) \times \prod_{k=1}^L \Phi_{\tilde{\mathcal{T}}_{G,k}|\mathcal{H}_0}(w_k t), \qquad (55)$$

we can write:

$$\Phi_{\tilde{y}|\mathcal{H}_0}(t)$$
$$= \left[\prod_{k=1}^L (1 - 2jw_k h_k t)\right]^{-\frac{1}{2}} \exp\left(\frac{-t^2}{2} \sum_{k=1}^L w_k^2 \sigma_k^2\right)$$
$$\times (1 - 2jw_0 t)^{-\frac{1}{2}}.$$

$$(56)$$

Since $P_{fa} = \text{Pr}\{\tilde{y} > \tilde{\eta}|\mathcal{H}_0\} = 1 - \mathcal{F}_{\tilde{y}|\mathcal{H}_0}(\tilde{\eta})$, we numerically invert the CF of $\tilde{y}$ under $\mathcal{H}_0$ using the method introduced in [25]. This technique approximately calculates the distribution function of a standardized random variable, when its characteristic function is known. Following the strategy discussed in [25], we first define the standardized test statistic

$$Z \triangleq \frac{\tilde{y} - E_0[\tilde{y}]}{\sqrt{\text{Var}_0[\tilde{y}]}}, \qquad (57)$$

so the approximate distribution function of Z can be obtained as:

$$\hat{\mathcal{F}}_{Z|\mathcal{H}_0}(z) \simeq \frac{1}{2} + \frac{\xi z}{2\pi} - \sum_{\substack{\upsilon=1-H \\ \upsilon \neq 0}}^{H-1} \frac{\Phi_{Z|\mathcal{H}_0}(\xi \upsilon)}{2\pi j \upsilon} e^{-j\xi \upsilon z}, \qquad (58)$$

where ξ is a constant variable which ensures that the full range of $\hat{\mathcal{F}}_{Z|\mathcal{H}_0}(z)$ is considered (i.e. it includes 0 and 1) and H defines the number of points used in the approximation of CDF. The values for z may be chosen as the Fourier frequencies, that is, $z_k = (2\pi(k - H)/2\xi(H - 1))$ for $k = 1, 2, \cdots, 2H - 1$. The summation in the approximation formula (58) can be computed using the fast Fourier transform, if the $\upsilon \neq 0$

term is subtracted from the FFT result. Furthermore, the characteristic function $\Phi_{Z|\mathcal{H}_0}(.)$ in (58) can be calculated as follows:

$$\Phi_{Z|\mathcal{H}_0}(t) = e^{\frac{-j(w_0 + \Sigma_{i=1}^L w_i h_i)}{(w^T m_0 w)^{1/2}}} \times \Phi_{\tilde{y}|\mathcal{H}_0}\left((w^T m_0 w)^{-1/2} t\right), \tag{59}$$

Finally, the intended distribution function can be obtained as

$$\hat{\mathcal{F}}_{\tilde{y}|\mathcal{H}_0}(\tilde{\eta}) \simeq \hat{\mathcal{F}}_{Z|\mathcal{H}_0}\left((w^T m_0 w)^{-\frac{1}{2}}(\tilde{\eta} - w_0 - \sum_{i=1}^L w_i h_i)\right), \tag{60}$$

Therefore, the global decision threshold in (41) can be analytically determined as

$$\tilde{\eta} \simeq \hat{\mathcal{F}}_{\tilde{y}|\mathcal{H}_0}^{-1}(1 - P_{fa}). \tag{61}$$

5. Simulation Results and Discussions

In this section, we evaluate the performance of the proposed CSS method. The simulated primary signal is a DVB-T ((Digital video broadcasting, Terrestrial television) signal with 64-QAM subcarrier modulation. Following the settings in DVB standard, we set the values of FFT length, number of occupied channels, and the length of guard interval as $N_{FFT} = 8192$, $N_{occ} = 6817$, and $N_g = 1024$, respectively. The transmission mode is selected to be 8K mode and carrier frequency is set to 750 MHz. It is assumed that Pfa=0.01, L=10 and the sensing duration is 3 OFDM symbols.

It is well-known that the peaks of the cyclic autocorrelation function of an OFDM signal occurs at $\tau = \pm N_{FFT}$ and $\alpha = k/(N_{FFT} + N_g)$ for integer k.

In all single-cycle simulations, the SU employs $\alpha = 1/(N_{FFT} + N_g)$ and $\tau = N_{FFT}$. Furthermore, the cyclostationary detectors use a Kaiser window with length $P = 2048$ and $\beta = 10$. The average signal-to-noise ratio (SNR) in the i th observation channel is defined as $SNR_i = 10\log_{10}(\tilde{\delta}_x^2/\delta_i^2)$, where $\tilde{\delta}_x^2$ denotes the variance of the PU's signal.

5.1 Local Sensing

Fig. 3 presents the receiver operating characteristics (ROC) curves for the non-cooperative spectrum sensing over frequency-flat Rayleigh fading channel. Analytical curves are obtained from (29). It is evident that, for a sufficiently large sample size, the numerical results follow very closely the theoretical curves.

The accuracy of the analytical null distribution (28) is examined in Fig. 4. As it can be seen, the simulated CDF under null hypothesis matched well

with the asymptotic analytical distribution. Therefore, analytically-computed
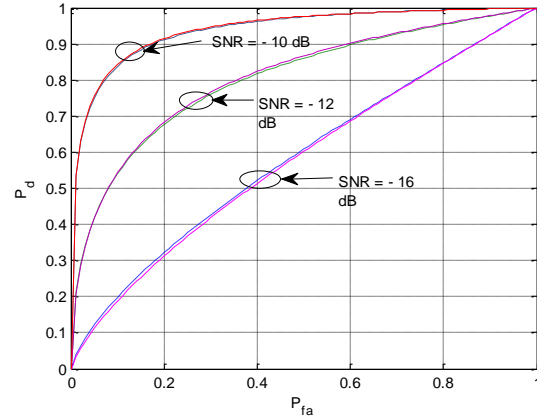

Fig. 3 ROC curves over frequency-flat Rayleigh fading channels (dashed lines: simulation, solid lines: analysis).
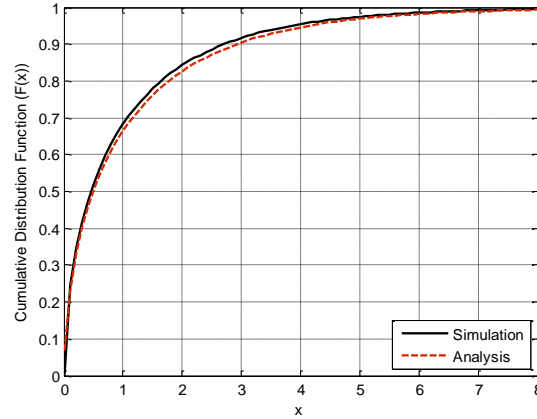

Fig. 4 Null distribution function of the proposed cyclic detector.

threshold values for a pre-defined false alarm rates are almost accurate.

In addition, performance comparisons between the proposed multi-cycle sensing method and some state-of-the-art competing methods are provided in Figs. 5 and 6. The simulated sensing methods are the conventional energy detector [3], well-known Lunden-Koivunen's (LK) cyclostationary detector [2], Chaudhari-Koivunen's autocorrelation-based detector [26], and Derakhshani-Le-Ngoc's cyclostationary detector [13].

As we can see, the proposed method has close detection performance to the LK detector which employs the full correlation structure of the second-order cyclic moment. Since the proposed test statistic has lower computation complexity compared to the LK algorithm, it can be considered as a potential substitute.
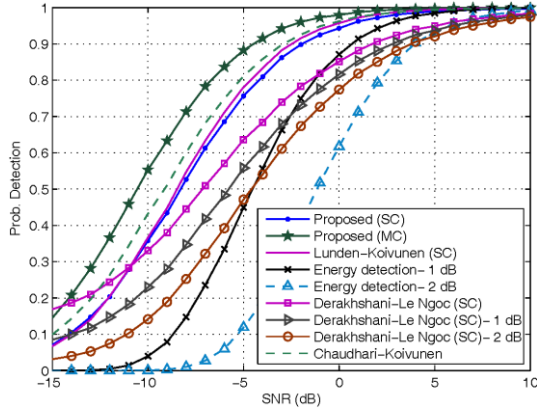
Fig. 5 Detection performance of proposed multi-cycle detector over frequency-flat Rayleigh fading channel (fd=150 Hz and Pfa=0.01).
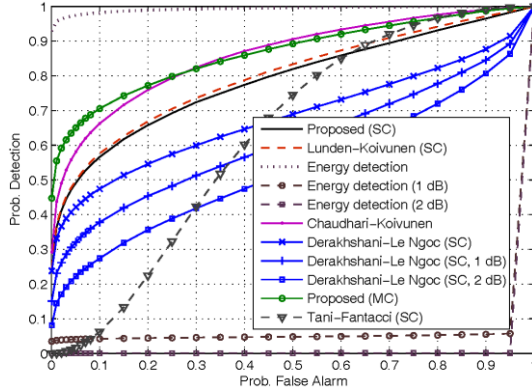

Fig. 6 ROC curves over Rayleigh fading channels with average SNR = -10 dB and fd=150 Hz.

Furthermore, the single-cycle cyclostationary detector of [13] uses the following hypothesis test:

$$\mathbb{T}_{\text{DL}} = \left| \widehat{\mathcal{M}}_{2x}(\alpha; \tau = 0) \right|^2 \overset{\mathcal{H}_1}{\underset{\mathcal{H}_0}{\gtrless}} \eta_{\text{DL}}. \tag{62}$$

where the associated threshold, which is set so the target Pfa is met, can be obtained as

$$\eta_{\text{DL}} = -\frac{2\delta^4}{M^2} \log P_{\text{fa}}. \tag{63}$$

In the above equation, δ^2 denotes the variance of complex AWGN channel. To illustrate the impact of noise uncertainty, assume $\delta^2 = \zeta\delta_n^2$, where δ_n^2 is the nominal noise power and $\zeta > 1$ is a parameter that specifies the size of the uncertainty [27]. Then, for a target Pfa, the threshold is $\eta_{\text{DL}} = \zeta^2\eta_{\text{DL,n}}$, where $\eta_{\text{DL,n}}$ is the true nominal threshold value. This problem causes a loss in detection performance. As it is evident in Figs. 5 and 6, the performance of $\mathbb{T}_{\text{DL}}$ is significantly degraded in the presence of an uncertainty in the noise power estimation. Consequently, in realistic applications, our proposed method provides much better detection performance for OFDM signals as compared to the cyclostationary detector of [13].

We also study the performance of low complexity cyclostationary detection method of [4]. The decision statistic of this method is

$$\mathbb{T}_{\text{TF}} = \left| \frac{\widehat{\mathcal{M}}_{2x}(\alpha; \tau)}{\widehat{\mathcal{M}}_{2x}(\alpha + S; \tau)} \right| \overset{\mathcal{H}_1}{\underset{\mathcal{H}_0}{\gtrless}} \eta_{\text{TF}}, \tag{64}$$

where

$$\eta_{\text{TF}} = \tan\left\{\frac{\pi(1 - P_{\text{fa}})}{2}\right\}. \tag{63}$$

The detection performance of $\mathbb{T}_{\text{TF}}$ is assessed in Fig. 6 through numerical simulation. The result reveals that our proposed multi-cycle sensing method outperforms this detector, as well as the other competing methods.

5.2 Cooperative Sensing Without Direct Observation at FC

In this subsection, it is assumed that there are 14 SUs in secondary network, and the vector of reporting channel variances is set to σ = [0.9, 1.2, 0.6, 2.2, 0.7, 1.5, 1.0, 0.7, 1.8, 0.8, 0.9, 1.5, 1, 2]T. In each simulation run, the fading coefficients of reporting channels are estimated for use in weight vector computation.

The accuracy of the proposed null distribution (i.e. (53)) is investigated in Fig. 7. As we can see, the analytical CDF follows very carefully the empirical accurate CDF. Furthermore, ROC curves for the proposed method as well as the conventional equal-gain combining (EGC) method are shown in Fig. 8. As it is evident, the proposed method outperforms the well-known EGC method of [2].
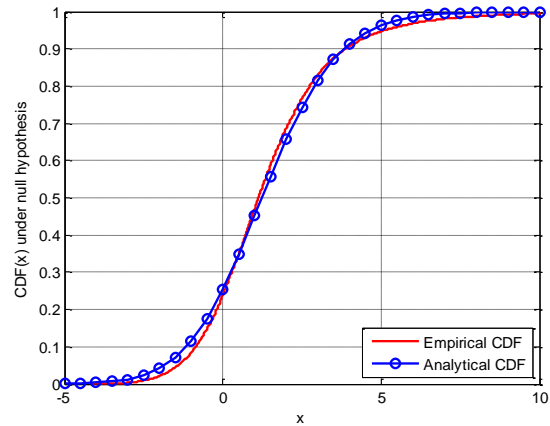

Fig. 7 Analytical versus simulated CDF for the null distribution in (53).
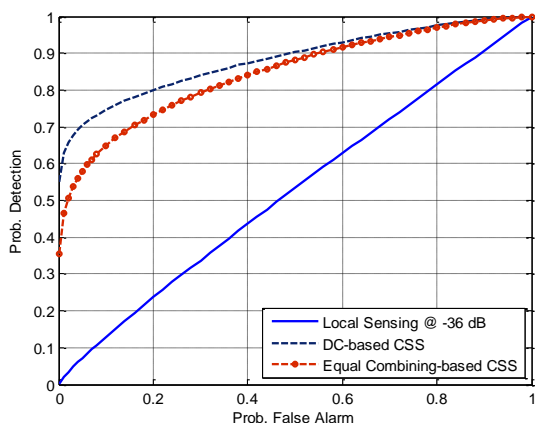
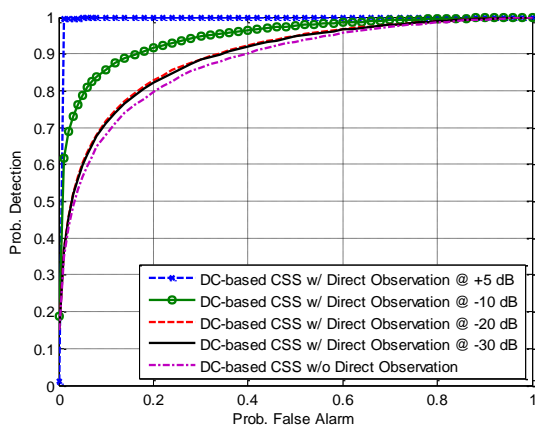Fig. 8 ROC curves over frequency-flat Rayleigh fading channel (fd=150 Hz).

Fig. 9 ROC curves for the cases where the FC has direct sensing capability, as compared to the conventional case. The sensing channels are assumed to be under frequency-flat Rayleigh fading channel (fd=150 Hz).

5.3 Cooperative Sensing With Direct Observation at FC

Performance of proposed CSS with FC's direct sensing capability assumption is evaluated in Fig. 9. It is assumed that there are 10 SUs in

the secondary network, and the vector of reporting channel variances is set to $\sigma = [0.9, 1.2, 0.6, 2.2, 0.7, 1.5, 1.0, 0.7, 1.8, 2.8]T$. As it is evident, if the FC has the ability to perform direct spectrum sensing, the global probability of detection can be improved. This capability can greatly increase the reliability of secondary network, even if the sensing channel of FC has very low SNR.

6. Conclusions

We proposed a spectrum sensing method based on properties of the second-order cyclic moment, and showed that this local sensing method significantly outperforms the conventional energy detector. In order to be able to perform statistical test, the null and alternative distributions of the proposed method are derived and verified through extensive numerical simulations.

Based on the proposed primary user detection method, we then developed a cooperative spectrum sensing (CSS) scheme. Numerical simulations show the advantageous of the proposed CSS over the widely-accepted equal-gain combining method of [2]. In addition, we proposed some local and global analytic threshold setting methods. Illustrative results confirm that the proposed methods provide enough accuracy.

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